ESTIMATION OF LYAPUNOV EXPONENTS FOR QUASI-STABLE ATTRACTORS OF DYNAMICAL SYSTEMS WITH TIME DELAY

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Lyapunov exponents play an important role among the invariant characteristics of dynamical systems. Analysis of the spectrum of Lyapunov exponents are widely used to study the complex dynamics in systems of ordinary differential equations and models that can be reduced to the maps. As follows from the Oseledets theorem in the finite-dimensional case the linearized on attractor system of the ordinary differential equations is always Lyapunov proper and thus the upper limit may be replaced by a normal limit, that allowing to effectively compute Lyapunov exponents. In this paper we consider the question of numerical evaluation of Lyapunov exponents for delay differential equations. The Oseledets theorem not proved in this case. We used a new algorithms with FFT, tested it on Hutchinson equation with known Lyapunov spectra and compared with results of old algorithms. The so-called quasi-stable behavior is typical for a number of gene networks models and neuronal associations that have been studied recently. The phenomenon of cycle (k-dimensional torus) quasistability in dynamical system is characterized by the fact that some of its multipliers are asymptotically close to the unit circle, and the remaining multipliers are modulo less than one (with the exception of a simple unit (k units)). In some cases, it is possible to prove the existence and give an asymptotic estimate of the studying system multipliers by using the large parameter method. However, if the large parameter methods are not applicable, it is necessary to obtain a tool for the numerical estimation of multipliers. Such a tool is provided by algorithms for Lyapunov exponents estimation. Given that the equations with delay are often applied in the models of neural and gene networks, the algorithm for Lyapunov exponents estimation for such systems will be in demand.