# BIFURCATIONS OF ZERO BALANCE STATE IN ONE BOUNDARY-VALUE PROBLEM WITH DEVIATION IN EDGE CONDITION

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Let us consider nonlinear boundary-value problem:

$$u' = \ddot{u} + \gamma u - u^3 \tag{1}$$

with edge conditions

$$u'(0,t) = 0, \tag{2}$$
$$u'(1,t) = \alpha u(x_0,t),$$

where parameters $\alpha, \gamma \in \mathbb{R}$, $x_0 \in [0,1]$. Equally with boundary-value problem (1), (2) let us consider the following system of differential equations

$$\dot{u_j} = N^2\left(u_{j-1} - 2u_j + u_{j+1}\right) - \gamma u_j - u_j^3, \qquad j = \overline{1,N}, \tag{3}$$

which simulates this problem. In this case edge conditions substitute for

$$u_0 = u_1, \tag{4}$$
$$u_{N+1} = u_N + \frac{\alpha}{N}u_k,$$

where $k \in [1,N]$ is determined by value $x_0$ in edge condition (2).

Our task of research was to find critical values of parameter $\alpha_{cr}(\gamma)$, when in system (3) with conditions (4) the trivial solution of boundary-value problem (1), (2) changes its stability. This task can be researched by two ways. One of these ways consists in the construction of characteristic equation, which can be obtained by means of Euler substitution for boundary-value problem (1), (2). As a result of this substitution there can be given a complicated transcendental equation. For this equation there can be found values $\alpha$, when all roots are in the left part of complex plane and one pair of them are on the imaginary axis. In this case there will be the loss of stability for trivial solution of boundary-value problem (1), (2). The proof of this location of eigenvalues for linearized equation (1) is quite difficult. So the only way of research consists in the solving of linearized system of ordinary differential equations (3) with conditions (4). For this system values $\alpha$ are selected according to stability or instability of zero balance state.

The numerical research was carried out by means of special software. All calculations were performed on a large number of independent streams. So the program uses technologies of parallel calculations.

As a result of numerical research there were found areas of values $\gamma$ and $\alpha$. For each of these areas there were researched the stability of zero balance state for boundary-value problem (1), (2).

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## Список литературы

[1] S.A. Kashchenko. About bifurcations at small perturbations in a logistic equation with delay // Model. Anal. Inform. Sist., v. 24, №2, pp. 168–185 (2017).