Structure and Dynamics of Two-Color Random Networks with Tree-Particle Interactions

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We consider equilibrium and relaxation properties of an ensemble of exponential random graphs with vertices bearing a quenched binary variable (color). The cost function (Hamiltonian) of the ensemble implies that there is a three-point interaction between vertices. Namely, it is linear in the number of "triplets" of the same color in the system, i.e. sequences of three vertices of the same color connected with edges, with the cost of a single triplet μ being a parameter of the system.

We start with a completely random dense Erdos-Renyi graph, assign colors to the vertices, and dynamically rewire the graph edges until the equilibrium state is reached. The dynamical rules we use are based on Metropolis algorithm coupled to the cost function described above, they conserve the total number of vertices and edges as well as total degree of each vertex. Two different variations of the dynamic rules are used: the so-called Maslov-Sneppen rules, and a novel generalization of these rules which we propose here in order to avoid dynamical trapping of the system.

In the previous studies of this system it was suggested that there exists a wide range of parameter μ (a so-called plateau) in which the equilibrium number of multicolor edges stabilizes at some non-zero value which is μ-independent. The aim of this work was to study the mechanism responsible for the formation of this plateau, in particular in order to understand whether it is a true equilibrium phenomenon or a result of quenching of the system in some mestastable state. Instead, we have been able to show that the plateau does not actually exist, and in the relevant range of μ the system eventually relaxes to the state where the number of multicolor edges is essentially zero. We show, however, that the relaxation is algebraically slow both in the case of standard Maslov-Sneppen dynamic rules and in the case of generalized rules proposed here. Finally, we discuss possible systems in which Maslov-Sneppen and generalized dynamics may produce different results.

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