Universal Level Statistics of the Out-of-Time-Ordered Operator

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The out-of-time-ordered correlator (OTOC) has been proposed as an indicator of chaos in quantum systems due to its simple interpretation in the semiclassical limit. In particular, its rate of possible exponential growth at $\hbar \to 0$ is closely related to the classical Lyapunov exponent. Here we explore how this approach to quantum chaos relates to the random-matrix theoretical description. To do so, we introduce and study the level statistics of the logarithm of the out-of-time-ordered operator, $\hat{\Lambda}(t) = \ln\left(-\left[\hat{x}(t), \hat{p}(0)\right]^2\right)/(2t)$, that we dub the "Lyapunovian" or "Lyapunov operator" for brevity. The Lyapunovian's level statistics is calculated explicitly for the quantum stadium billiard. It is shown that in the bulk of the filtered spectrum, this statistics perfectly aligns with the Wigner-Dyson distribution. Our results show that the Lyapunov operator may serve as a useful tool to characterize quantum chaos and in particular quantum-to-classical correspondence in chaotic systems, by connecting the semiclassical Lyapunov growth at early times, when the quantum effects are weak, to universal level repulsion that hinges on strong quantum interference effects.