Quasi-Stable Solutions of the Genetic Networks Models

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The simplest genetic oscillator, known as a repressilator, consists of three elements each unidirectionally inhibiting its neighbor. More specifically, the first of them inhibits the synthesis of the second, the second inhibits the synthesis of the third, and the third, which closes the cycle, inhibits the synthesis of the first one.

The interaction of the protein concentrations and of mRNA (message RNA) concentration is surprisingly similar to the interaction of six ecological populations – three predators and three preys. This new mathematical model is represented by a system of unidirectionally coupled ordinary differential equations. The existence and stability of special periodic motions (traveling waves) for this system is studied. It is shown that, with a suitable choice of parameters and an increasing number m of equations in the system, the number of coexisting traveling waves increases indefinitely, but all of them (except for a single stable periodic solution for odd m) are quasi-stable. The quasi-stability of a cycle means that some of its multipliers are asymptotically close to the unit, while the other multipliers (except for a simple unit one) are less than unity in absolute value.