Dynamics of qubits in the field of unipolar impulses: Magnus propagator, generalized "area theorem" and motion on groups

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As is well known, that the number of sequential operations over the register of superconducting qubits is essentially limited by the decoherence time in (~ 100 μs). One of the ways to overcome this limitation can be the implementation of "rapid quantum logic" for controlling qubits, which will allow manipulating quantum states with characteristic times of ~ 10 ps [1].

We discuss here the problem of accelerating quantum computations by controlling the qubit states with short unipolar pulses with a large amplitude - fluxons. A new method for describing the qubit dynamics based on the Magnus representation for the evolution operator (propagator) of qubits is proposed. For the transitions amplitude between stationary states at the time intervals that are shorter than the decoherence time, we will solve the time Schrodinger equation for calculating the evolution operator $U(t)$ - the propagator of the system, - which determines its wave function at any time: $\psi(t) = U(t)\psi(0)$, where $\psi(0)$ is the initial state. The usual perturbation theory is not applicable for our purposes, since it is necessary to consider signals of large amplitude for obtaining appreciable changes in the level population. To calculate the propagator $U(t)$ of an n-qubit system, we use two ideas: i) The Magnus expansion [2] for the propagator in the form of an exponential from some operator $M(t)$ (the "Magnus" operator): $U(t) = \exp(-iM(t))$, for which there is a simple computational algorithm in the form of a series in powers of the Hamiltonian with any required accuracy; ii) If in some basis the Hamiltonian for n -qubits is represented by a matrix size $N \times N$, where $N = 2n$, then the оператор $M(t)$ will also have dimension $N \times N$. Consequently, by applying the well-known Cayley-Hamilton theorem, the propagator in this case can be written in the form of an expansion in powers of the matrix, and the maximum degree of the resulting polynomial is not higher $N-1$: $U(t) = u_0 I + u_1 M(t) + u_2 M^2(t) + \cdots + u_{N-1} M^{N-1}(t)$. The coefficients of the expansion u_j for all $j = 0,1,\cdots N-1$ could be explicitly expressed in terms of the eigenvalues of the matrix $M(t)$. Note that from the point of view of group theory, we are talking about the decomposition of the group exponent over the matrices of the group. The evolution of the state of the system can be represented as a motion of the qubits polarization on such a group, similar to the rotation of the Bloch vector on the SU(2) group in the case of one qubit. As an example, we will demonstrate the effectiveness of the obtained expression for describing the dynamics of one qubit under various excitation methods by unipolar pulses. From theory of two-level system we know that in the periodically modulated pulse the Rabi oscillations occur as a function of the total interaction area of the pulse. This statement is called the "area theorem". An analog of the generalized "area theorem" is found in the case of unipolar impulses, which in a certain sense acts like the area theorem in the case of Rabi excitation.

The resulting expression for the propagator is then used to describe the influence of the pulses on a two-qubit system. First, a symmetrical configuration is considered when the qubit parameters are not different and they have the same field on the fluxon side. In this case, the matrix $M(t)$ has dimension 3×3 and belongs to the group SU(3). A three-level system of a more general type is also discussed here, which arises, for example, in the case when it is possible for one qubit to move to the higher layer under the influence of a fluxon. Analytical consideration is carried out for two qubits - a four-level system (the SU(4) group), - although the developed algorithm is applicable to any number of qubits (with allowance for computational constraints). To control the Magnus approximation, we use direct numerical simulation of the dynamics of a multi-qubit system, and as a criterion for the proximity of operations - the "degree of inconsistency" (fidelity) is used.

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