Cyclostationary models for simulation of non-stationary random processes

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No one can argue that the vast majority of processes under observation are not completely deterministic. The theory of random processes provides researchers with many basic models for their description and with rather developed techniques for their processing. As a matter of fact, the most popular and easily understood case is still a wide-sense stationary noise superimposed by the sum of sinusoids with deterministic or random parameters which remain constant at least within the observation frame. This process could be expressed in terms of its correlation function or power spectrum density as the Fourier counterpart. While the periodicity contained in the process is evident and could be extracted by a direct subtraction, the reminder of the process demonstrates time-limited statistical interdependency of any pair of its samples.

However, the stationary model would not be the best choice in the case of a process generated by more complex phenomena or changing in time environment. A possible solution here is the use of a set of locally stationary models or one yet slowly changing model as it is claimed by Kalman's and other related filtering methods. The reliable alternative to that solution could well be the global probabilistic model of the random process with the hidden periodicity which would take into account the regular change of statistical properties. Although having started in the 1950s, not so long after N. Weiner gave exhausted description of the stationary case, research into the hidden periodicities of the second order was prolific, the main results that has been obtained over few decades are generally not familiar to the significant majority of modern researchers.

The cyclostationarity, a term originally introduced by W. Bennet to describe digitally modulated pulses, generalizes the concept of the hidden periodicity including but not limited to the prominent case of periodically correlated time series and continuous signals. Not only is the cyclostationarity a natural yet not evident expansion of the stationarity, but any stationary process could be considered as a particular case of cyclosationary (CS). It is possible due to the main advantage of the CS approach which is that the characteristics of the well-known stationary models simply become the essential and additive part of the corresponding characteristics of the CS models.

The simplest yet most important and easy to understand example of wide-sense CS process. As it follows from its name, the first and second order probabilistic characteristics will be enough to express all the properties of the process. In fact, the first order or the mean function are quite the same as for the stationary case. The essential part standing the CS processes out against other types is the behavior of its two-dimensional, or dyadic, correlation function. Actually, the latter could be expanded into a Fourier series with respect to the current time in the case of strict periodicity or into generalized harmonic Fourier series in the more complex almost-periodical case whereby the frequencies are not rationally commensurable.

On the whole, originally introduced in the electrical engineering, cyclostationarity provides researchers with useful models that could be adapted to a process of any nature, i.e., physical, economic, biological, where the hidden periodicity is expected but could not be found explicitly regardless to how long the sample was observed.