Numerical studies of solutions for kinetic equations with many-particle collisions

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In this work we present use of low-rank tensor decompositions for acceleration of evaluation of right-hand side of systems of kinetic equations with many-particle collisional terms. These equations can be interpreted as a generalization of classical Smoluchowski aggregation equations allowing one to consider not only binary collisions of particles but also triple particle collisions.

Straight-forward evaluation of right-hand side for such system of N equations with $k = 1, 2, \ldots, N$ requires $O(N^3)$ numerical operations and we find such complexity too high for practical investigations. However, under assumptions that the kinetic coefficients can be represented with either canonical polyadic (CP) or tensor train decomposition (TT) with rank $R \ll N$ we can propose algorithms evaluating the right-hand side with much lower complexities: $O(NR \log N)$ and $O(NR^2 \log N)$ for CP and TT respectively.

We check the accuracy of proposed approach for model Cauchy problem with constant kinetic coefficients and monodisperse initial conditions and obtain good agreement of numerical results with known explicit solution. With use of our ideas we reach high level of accuracy of numerical solutions in really modest CPU-times. We compare numerical solutions with different triple-collision rates and obtain a significant influence of accounting triple collisional effects.