Statistical Mechanics of Money

Victor M. Yakovenko
Department of Physics, CMTC and JQI, University of Maryland, College Park, USA

By analogy with the probability distribution of energy in statistical physics, the probability distribution of money $P(m)$ among the agents in a closed economic system is expected to follow the exponential Boltzmann-Gibbs law [1]. This conjecture is confirmed by agent-based computer simulations [2]. We start from the state of perfect equality, where the total money M is equally divided among N agents, so the initial probability distribution is $P_0(m) = \delta(m - M/N)$. Then, certain amounts of money Δm are repeatedly transferred from one randomly selected agent to another in payment for products and services. If the selected agent does not have enough money to pay, we skip the transaction and go to another pair of agents. As time goes on, the initial delta-function distribution broadens and eventually converges to the equilibrium exponential distribution $P(m) = \exp(-m/T)/T$ with the money temperature $T = M/N$. This Boltzmann-Gibbs distribution represents "natural" inequality spontaneously developing due to entropy maximization in the randomized free-market model of small producers, which was extensively discussed by Karl Marx in *Das Kapital*. Empirical data on money distribution are difficult to obtain, but plenty of data are available on income distribution. Data analysis shows that income distribution in the USA [3], European Union [4], and many other countries [5] has a well-defined two-class structure. The majority of the population (about 97%) belongs to the lower class characterized by the "thermal" exponential distribution, whereas about 3% of the population are in the upper class characterized by the Pareto power-law ("superthermal") distribution, and there is no evidence for "middle class". The share of the total income going to the upper class expands dramatically during bubbles in financial markets and contracts during crashes [6]. This redistribution of income between the lower and upper classes is responsible for the overall dynamics of inequality in the last 30 years. For more information, see a popular article [7] in the special issue of the *Science* magazine on *The Science of Inequality* and the Web page `http://physics.umd.edu/~yakovenk/econophysics/`.

References

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